

Calculus 1

Midterm Exam

October 3, 2022 (18:30 – 20:30)



university of
 groningen

Please read the instructions!

1) Prove using the ε - δ definition that $\lim_{x \rightarrow 3} (4x - 1) = 11$.

2) Find the following limits without applying l'Hospital's Rule.

a) $\lim_{x \rightarrow -2} \frac{x^2 - 3x - 10}{x + 2}$

b) $\lim_{x \rightarrow 0} \frac{\sqrt{\cos(x)} - 1}{x^2}$

c) $\lim_{x \rightarrow \infty} \frac{e^x - e^{-x}}{e^x + e^{-x}}$

3) Show using the Squeeze Theorem that $\lim_{x \rightarrow 0} \sqrt{x^2} \cos\left(\frac{1}{x^2}\right) = 0$.

4) Prove the following statement using mathematical induction on N .

“(1 + x)^N ≥ 1 + Nx for every integer N ≥ 0 and real number x > -1.”

5) Find the derivative of the real function $f(x) = e^{\left(\frac{x}{x^2+1}\right)}$. At each step, indicate which rule of differentiation is being applied.

6) Use Implicit Differentiation to obtain an equation of the tangent line to the ellipse $3x^2 + 2y^2 = 11$ at the point $(-1, 2)$.

7) Suppose f is an odd function and is differentiable everywhere. Prove that for every number $b > 0$, there exists a $c \in (-b, b)$ such that $f'(c) = f(b)/b$.

8) Apply l'Hospital's Rule to find the following limit: $\lim_{x \rightarrow 0} \left[\cos(x) \right]^{\frac{1}{\sin(x)}}$.

9) Determine the extrema (max/min) of the real function $f(x) = xe^{-x^2}$.

Instructions

- **write your name and student number on the envelope and on the top of each sheet of writing paper!**
- use the writing (lined) and scratch (blank) paper provided, raise your hand if you need more paper
- start each question on a new page
- use a pen with black or blue ink
- do not use any kind of correcting fluid or tape
- any rough work should be crossed through neatly so it can be seen
- this exam is open-book, you may use the textbook or the lecture notes
- you are allowed to use a simple pocket calculator
- programmable/graphing calculators are not allowed, nor the use of electronic devices (tablet, laptop, phone, etc.) to solve the exercises
- your work should be clearly and logically structured
- **explain your reasoning using words**
- show all your calculations, an answer without any computation will not be rewarded
- each problem is worth 10 points
- upon completion¹ place your worksheets in the envelope and submit them at the front desk

¹At the end of the exam or after you finished whichever is sooner.